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PAT-003-1162004 Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

August / September - 2020

Mathematics : Course No. - 2004

(Methods in Partial Differential Equation)

(New Course)

Faculty Code : 003

Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries **14** marks.

1 Answer any **seven** : (Each question carries **two** marks) **14**

(a) Define : (i) Complementary function and (ii) Singular solution.

(b) Solve $\left(D^3 - 7D'^2 + 6D'^3\right)Z = 0$.

(c) Verify the equation $z = \sqrt{zy+b} + \sqrt{2x+a}$ is the solution of $z = \frac{1}{p} + \frac{1}{q}$.

(d) State Lipchitz condition for the functions of three variables (x, y, z) from the point (a, b, c).

(e) Find the complete integral of $p^3 + q^3 = 3$.

(f) Find the direction cosines of the normal to the surface $4x - 6y - 10z = 7$ at the point (2, 1, 1).

(g) Verify the equation is exact or not $\left(y^2\right)dx + \left(x^2\right)dy + \left(3x^2\right)dz = 0$.

(h) Solve $f\left(x^2 + y^2 + z^2,\ z^2 - 2xy\right) = 0$.

(i) Determine the envelope of the two parameters system of surfaces $(x-a)^2 + (y-b)^2 + z^2 = 1$.

(j) Define : (i) Tangent plane and (ii) Pffafian Differential form.

2 Answer any **two** of the following : **2x7=14**

(a) Find the general solution of

$$(D-D')(D+D')z=e^{2x-y}(x+2y).$$

(b) Find the primitive solution of

$$2y(a-x)\,dx+\left[\left(z-y^2\right)+(a-x)^2\right]dy-y\,dz=0.$$

(c) If $(\alpha D+\beta D'+\gamma)^n$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')Z = 0$ is,

$$z=e^{\frac{-\gamma}{\alpha}x}\left(\phi_1(\beta x-\alpha y)+y\phi_2(\beta x-\alpha y)+.......+y^{n-1}\phi_n(\beta x-\alpha y)\right).$$

Where $\phi_i=\phi_i(\varepsilon)$ is an arbitrary function of a single variable $(i=1,2...,n)$.

3 All are compulsory : **14**

(a) Solve using Nattani's method **5**

$$2yz\,dx-2xz\,dy-\left(x^2-y^2\right)(z-1)\,dz=0.$$

(b) Find the particular integral of **5**

$$\left(D^2+2D'^2-2DD'\right)z=\cos(x+y).$$

(c) Solve the equation $\dfrac{dx}{(x+z)}=\dfrac{dy}{(y)}=\dfrac{dz}{\left(z+y^2\right)}$. **4**

OR

3 All are compulsory : **14**

(a) Prove that for any non-zero functions $\mu=\mu(x,y,z)$ and **5** $X=(P,Q,R)$ where *P, Q, R* are the functions of x,y,z then $X\cdot Curl\,X=0\ Iff\ (\mu X)\cdot curl(\mu X)=0$.

(b) Find the equation of the system of curves on the cylinder $2y=x^2$ orthogonal to its intersection with the hyperbola of one-parameter system $xy=z+c$. **5**

(c) Find the integral curves of the equation **4**

$\frac{dx}{(cy-bz)}=\frac{dy}{(az-cx)}=\frac{dz}{(bx-ay)}$ and show that they are circles.

4 Answer the following both : **2x7=14**

(a) Solve the partial differential equation

$$px\left(z-2y^2\right)=(z-qy)\left(z-y^2-2x^3\right).$$

(b) Determine the partial differential equation from the relation $F(u,v)=0$, where u and v are functions of x, y and z, with z is dependent of x and y.

5 Answer any **two** of the following : **2x7=14**

(a) Describe Jacobi's method.

(b) Classify the equation and convert it in canonical form $2r-5s+3t=x$.

(c) Solve $2(z+xp+yp)=yp^2$ using Charpits's method.

(d) Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curve of intersection with the plane parallel to plane $X-Y$.